

Geosystems Engineering Seminar

Monday, February 20th Rm. Mason 142A

Direct and Inverse Diffusion problems in Geomechanics

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Abstract

1. Subsidence Diffusion

Compaction of a collapsible substratum due to effective stress increase may give rise to the formation of the well-known trap-door mechanism (Terzaghi 1936, Vardoulakis et al. 1981). According to early works, large-scale subsidence over a yielding underground geostructure is seen as a stochastic (Markov) process (Litwinskiy 1974, Dimova 1990). In simple terms we assume that the displacement of a particle in vertical direction causes movement of particles mainly lying above it in a manner that particle vertical displacement is spread also horizontally. This process may be called *kinematic diffusion*, since it leads to the Einstein - Kolmogorov integral equation (cf. Tikhonov and Samarskii, 1963).

Under certain physical conditions and transformations of the coordinate system the E-K integral equation satisfies some partial differential equation of parabolic type, where vertical coordinate replaces time. The solution depends on a diffusivity coefficient, which is fitted in experimental results and determines the formation of the subsidence trough inside the body as well as in the surface. The plain strain and the axisymmetric models were concerned for simulating the trap door problem using Litwinskiy's theory of deep subsidence.

According to the geometry of well-known oil reservoirs (cf R.R. Boade et al, 1989), an elliptic model of deep subsidence is also presented. The bottom of the trap-door mechanism is assumed elliptic. The contour lines of subsidence are also ellipses, similar with the boundaries of the trap door bottom or confocal depending on the model. Results are compared with those of orthogonal base geometry as well as with plain and axisymmetric problems. Initial condition of the direct problem is the base subsidence and the solution yields the surface subsidence. The solution depends on a diffusivity coefficient, which determines the formation of the subsidence trough inside the body as well as on the surface.

This is the Direct Subsidence-Diffusion-Convection (DSDC) problem. Considering the results of the DSDC problem we present here the inverse SDC (ISDC) problem as well, using two kinds of regularization (cf. Lattés and Lions, 1969). In particular Lion's u_{xxxx} - method is compared to the presently proposed u_{xxxx} -method. Stability, in the sense of the von Neumann condition is ensured where the amplification factor depends on the regularization parameters ε and n . A first approach to convergence is done in the sense of the norm of the amplification factor. Another convergence study is given in terms of the truncation error (Richtmyer and Morton, 1967). Early well posedness of the initial and boundary value problems is examined in terms of linear stability analysis (Tomée, 1970). Well-posedness in terms of energy method is discussed in a recent study by Vairaktaris & Makridakis (2006).

2. Moisture Diffusion

Moisture diffusion in partially saturated media is also discussed in this presentation. Using continuum mixtures-theory approach to a three-phase medium (Vardoulakis & Sulem, 1995), we formulate the governing storage equation. The concept of suction in a partially saturated porous medium is outlined (Richards, 1931). On this basis we discuss the validity and the possible extension of Darcy's law in unsaturated flow conditions. This is done here by a qualitative analysis of the structure of the stationary saturation front in 1D-flow conditions. Finally the governing equations for unsaturated flow in terms of the volumetric water content are summarized. Numerical solution of the highly non-linear PDE uses an exponential diffusivity function based on experimental results (Philip J. R., 1969). Currently we study the influence of the dimensionless analysis of the in.-bound. value problem in the stability of the numerical algorithm.